Chapter 13

The Perfect Gas

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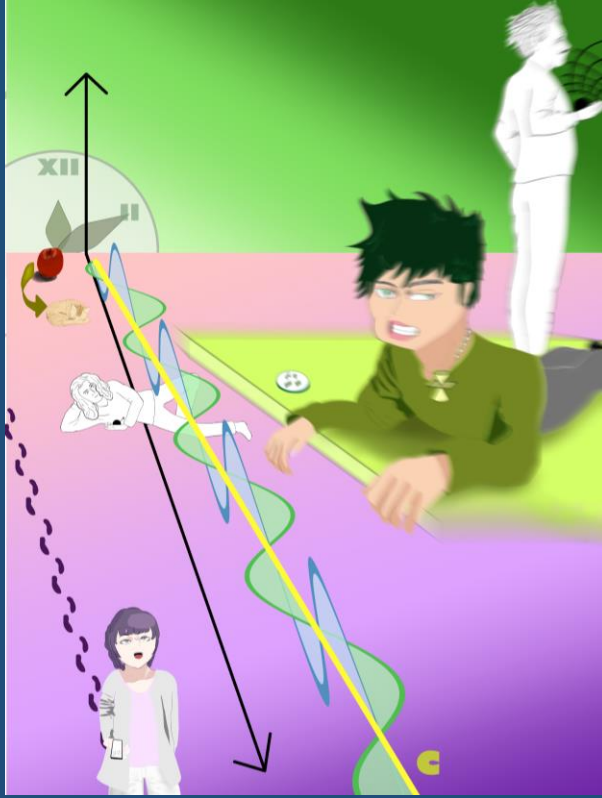
The Perfect Gas with Radiation



4章では、
理想気体だけを考えて熱力学量を考えていた。
13章では、
光の輻射圧も考慮に入れる。

$$P = P_{gas} + P_{rad} = \frac{\Re}{\mu}\rho T + \frac{a}{3}T^4$$

a: radiation density constant($= \frac{4}{c}\sigma$)

Bを全圧のうちのガスの割合として定義する。このとき

$$1 - \beta = \frac{P_{rad}}{P} = \frac{a}{3}\frac{T^4}{P}$$

$$\left(\frac{\partial \beta}{\partial T}\right)_P = -\left[\frac{\partial(1-\beta)}{\partial T}\right]_P = -\frac{4}{T}(1-\beta), \qquad \left(\frac{\partial \beta}{\partial P}\right)_T = -\left[\frac{\partial(1-\beta)}{\partial P}\right]_T = -\frac{1}{P}(1-\beta)$$

$$P = \frac{\Re}{\mu}\rho T + \frac{a}{3}T^4, \qquad \beta = \frac{\Re}{\mu}\frac{\rho T}{P}, \qquad 1-\beta = \frac{a}{3}\frac{T^4}{P}$$

$$\rightarrow \rho = \frac{\mu}{\Re}\frac{1}{T}\left(P - \frac{a}{3}T^4\right) = \frac{\mu}{\Re}\frac{P}{T}\beta$$

このとき

$$\alpha \equiv \left(\frac{\partial \ln \rho}{\partial \ln P}\right)_{T,\mu} = \frac{\mu}{\Re}\frac{P}{\rho T} = \frac{1}{\beta}$$

$$\delta \equiv -\left(\frac{\partial \ln \rho}{\partial \ln T}\right)_{P,\mu} = 1 - \frac{\mu}{\Re}\frac{P}{\rho}\left\{-\frac{4}{T}(1-\beta)\right\} = 1 + \frac{4(1-\beta)}{\beta} = \frac{4-3\beta}{\beta}$$

$$\varphi \equiv \left(\frac{\partial \ln \rho}{\partial \ln \mu}\right)_{T,P} = 1$$

単原子分子と光子の系で比熱c_Pと∇_{ad}を求めよう

準備：$\left(u_{rad} = \frac{P_{rad}}{3\rho}, nkT = \frac{\rho}{\mu}\Re T\right)$

$$u = \frac{3}{2}kT\frac{n}{\rho} + \frac{aT^4}{\rho} = \frac{3}{2}\frac{\Re}{\mu}T + \frac{aT^4}{\rho} = \frac{\Re T}{\mu}\left[\frac{3}{2} + \frac{3(1-\beta)}{\beta}\right]$$

$$\left(\frac{\partial u}{\partial T}\right)_P = \frac{\Re}{\mu}\left[\frac{3}{2} + \frac{3(1-\beta)}{\beta} + T\frac{-3\beta - 3(1-\beta)}{\beta^2}\left\{-\frac{4}{T}(1-\beta)\right\}\right]$$
$$= \frac{\Re}{\mu}\left[\frac{3}{2} + \frac{3(4+\beta)(1-\beta)}{\beta^2}\right]$$

$$\beta = \frac{\Re}{\mu}\frac{\rho T}{P}, \quad 1-\beta = \frac{a}{3}\frac{T^4}{P}$$

$$\left(\frac{\partial \beta}{\partial T}\right)_P = -\frac{4}{T}(1-\beta)$$

単原子分子と光子の系で比熱c_Pと∇_{ad}を求めよう

$$c_P=\left(\frac{\partial u}{\partial T}\right)_P-\frac{P}{\rho^2}\left(\frac{\partial \rho}{\partial T}\right)_P$$

$$=\frac{\Re}{\mu}\left[\frac{3}{2}+\frac{3(4+\beta)(1-\beta)}{\beta^2}\right]+\frac{PT}{\rho}\frac{4-3\beta}{\beta}$$

$$=\frac{\Re}{\mu}\left[\frac{3}{2}+\frac{3(4+\beta)(1-\beta)}{\beta^2}+\frac{4-3\beta}{\beta^2}\right]$$

$$\Rightarrow(\beta\to 1)\frac{5\Re}{2\mu},\qquad(\beta\to 0)\infty$$

$$\beta=\frac{\Re}{\mu}\frac{\rho T}{P},$$

$$\nabla_{ad}=\frac{\Re\delta}{\beta\mu c_P}=\frac{1}{\beta}\frac{4-3\beta}{\beta}\left(\frac{3}{2}+\frac{3(4+\beta)(1-\beta)}{\beta^2}+\frac{4-3\beta}{\beta^2}\right)^{-1}$$

$$=\left(1+\frac{(1-\beta)(4+\beta)}{\beta^2}\right)\Big/\left(\frac{5}{2}+\frac{(1-\beta)(4+\beta)}{\beta^2}\right)\Rightarrow\begin{cases}0.4(\beta\to 1)\\0.25(\beta\to 0)\end{cases}$$

adiabatic index γ_{ad}:

$$\gamma_{ad} \equiv \left(\frac{\partial \ln P}{\partial \ln \rho}\right)_{ad}$$

$$= \left\{\left(\frac{\partial \ln \rho}{\partial \ln P}\right)_T + \left(\frac{\partial \ln \rho}{\partial \ln T}\right)_P \left(\frac{\partial \ln T}{\partial \ln P}\right)_{ad}\right\}^{-1}$$

$$= \frac{1}{\alpha + \delta \nabla_{ad}}$$

$$\Rightarrow (\beta \to 1)\,\frac{1}{1 + \nabla_{ad}}, \qquad (\beta \to 0)\,\frac{4}{3}$$

adiabatic exponents:

$$\Gamma_1 \equiv \left(\frac{\partial \ln P}{\partial \ln \rho}\right)_{ad}, \qquad \Gamma_2 \equiv \left(\frac{\partial \ln P}{\partial \ln T}\right)_{ad}, \qquad \Gamma_1 \equiv \left(\frac{\partial \ln T}{\partial \ln \rho}\right)_{ad}$$